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We analyze a quantum effect that takes place in resonance-mediated many body decays. This effect is due to the interference between various contributions from a given intermediate resonant state, each one with a different virtual mass. We show that, although it can be safely neglected in most cases, the effect becomes strongly enhanced for some particular processes. We point out that these considerations have not been taken into account in the experimental study of D and B meson decays and could significantly affect several reported results.

Three or more body decays of heavy mesons are usually dominated by intermediate resonances. The latter are short lived states that cannot be directly observed: only daughter particles produced through their decays reach the detectors [1]. The detected final state results from the interference of all possible intermediate channels. Thus, one has to disentangle the quantum interference to understand the underlying physics. A powerful technique to split up the various resonant channels is the so-called Dalitz plot fit [2].

In general, if a given resonance proceeds within a kinematic region where no other resonances occur, interference effects are assumed to be negligible. In this case, the branching ratio of the heavy meson decay to that particular resonance can be measured in a simple way: one just counts the number of events in which the invariant mass of the decay products lies within a small window around the resonance central mass. The size of this window is chosen according to the width of the resonance. This is a natural and thus widely used measurement technique [3].

In this Letter we show that this method is not always safely applicable. In fact, the resonance is a virtual state, and its squared four-momentum can reach in principle any value within the allowed phase space. As we will see, in some particular cases the quantum interference arising from these states can be very important (notice that we are referring now to different intermediate states corresponding to *the same* resonance, while the quantum interference mentioned in the first paragraph above occurs between intermediate channels driven by *different* resonant states). We show here that the invariant mass of the detected particles can be indeed very far from the resonance mass shell, having proceeded yet through this particular resonance (with "far", we mean in comparison with the resonance width). Although this quantum effect is usually negligible, it can be strongly enhanced if the resonant decay is somehow suppressed at the resonance mass.

Let us describe those aspects of many body decays relevant to our discussion. Consider a heavy meson P decaying into a final state given by three detected particles, d_1, d_2 and d_3, and assume that the decay proceeds through intermediate resonances R_1, R_2, etc. [4]. The total amplitude of the decay is the sum of the amplitudes of all partial channels, each one mediated by a resonance R_i:

$$\mathcal{A}_{tot} = \Sigma_i \mathcal{A}_{R_i} \ . \tag{1}$$

For simplicity, let us assume that the decay is dominated by a single resonance R, in such a way that $\mathcal{A} \simeq \mathcal{A}_R = \mathcal{A}(P \to Rd_3; R \to d_1 d_2)$. It is natural to describe the process by considering three independent stages: resonance production $P \to Rd_3$, resonance propagation, and resonance decay $R \to d_1 d_2$. The amplitude factorizes then as

$$\mathcal{A}(P \to Rd_3 \to d_1 d_2 d_3) = \mathcal{A}(P \to Rd_3) \times BW_{R,12} \times \mathcal{A}(R \to d_1 d_2) \ . \tag{2}$$

The amplitudes $\mathcal{A}(P \to Rd_3)$ and $\mathcal{A}(R \to d_1 d_2)$ have to take into account the conservation of angular momentum in the decays, as well as the energy dependence, usually parameterized through form factors (we will come back to these two ingredients later). As usual, we describe the resonance propagation by means of a relativistic Breit-Wigner function [5]

$$BW_{R,12}(m_{12}^2) = \frac{1}{m_0^2 - m_{12}^2 - i m_0 \Gamma} \ , \tag{3}$$

where m_0 is the resonance mass, Γ is the resonance width, and m_{12}^2 is the invariant mass of the outgoing particles d_1 and d_2, $m_{12}^2 = (p_1 + p_2)^2$.

The function $|BW(m_{12}^2)|$ is peaked around the resonance mass, and decreases with a rate given by Γ. This behavior reflects the fact that R is a virtual particle that —according to quantum mechanics— can have *any* invariant mass m_{12}^2, the relative probability of each "mass" being weighted by the factor $|BW(m_{12}^2)|^2$. It is then natural to measure $BR(P \to Rd_3)$ by simply counting the number of detected particles d_1 and d_2 for which the value of $\sqrt{m_{12}^2}$ lies within a region of the order of $(m_0 - \Gamma, m_0 + \Gamma)$. This technique has in fact been used for many years [3]. However, as we will show in the following, the usage of this approach is not always safe.

The decay of a resonance is usually driven by the strong interaction. Thus, resonances have relatively large widths —some dozen or even some hundreds of MeVs [1]. However, if a particular decay is somehow suppressed, the resonance may have a longer life, and its width can be as small as some MeVs or less. This happens in particular when the resonance central mass m_0 is very close to the threshold of its decay to $d_1 d_2$. In this case, the phase space available for the decay turns out to be very narrow, and the (virtual) resonance could be allowed to decay through other channels which were in principle expected to be strongly suppressed in comparison with the "natural" strong channel $R \to d_1 d_2$.

This is the case, for instance, of the $\phi(1020)$ vector meson. For this resonance, the natural decay channel is $K\bar{K}$, in the same way as the natural decay for ρ is two pions. Nevertheless, due to the small phase space available —32 MeV and 24 MeV for the charged and neutral kaons, respectively— the corresponding ϕ branching ratios are "only" 49% and 34% for K^+K^- and $K\bar{K}$, respectively. Electromagnetic decays, that have branching fractions as small as 10^{-4} in the ρ decay pattern, are of the order of 1% in the ϕ case. Accordingly, the ϕ width is about 35 times smaller than the ρ one.

Other examples are low mass D^* vector mesons. Their natural decay channel is $D\pi$, in the same way as the natural decay for K^* is $K\pi$. But here, the phase space is as small as 7 MeV for the $D^{*0}(2007) \to D^0\pi^0$, 6 MeV for both $D^{*+}(2010) \to D^0\pi^+$ and $D^{*+}(2010) \to D^+\pi^0$, and 8 MeV for $D_s^{*+} \to D_s^+\pi^0$. As a consequence, measured branching ratios of these decays —which otherwise should reach almost 100%— are as "small" as 62%, 68%, 31% and 6%, respectively [6]. In the $D^{*0}(2007)$ decay pattern, the electromagnetic decay is as large as 38%, i.e., of the same order of magnitude of the strong one (in contrast, in the K^* case, electromagnetic decays are of the order of 10^{-3}). Accordingly, resonance widths are quite small: the width of $D^{*+}(2010)$ has recently been reported to be as small as 0.1 MeV [8], whereas for $D^{*0}(2007)$ and D_s^{*+} only upper limits are known, presently of the order of 2 MeV.

Let us face the study of heavy meson decays mediated by these particular spin one resonances, focusing on cases in which the detected final state includes their natural, and yet highly suppressed, strong channels. We describe here a usual situation [1], where both the initial heavy meson and the particles in the final state are scalars, $P = D, B$, and $d_i = \pi, K, D$. In this case, Eq. (2) can be conveniently written as

$$\mathcal{A}(P \to Rd_3 \to d_1 d_2 d_3) = F_{P,Rd_3}\, F_{R,d_1 d_2}\, (-2\vec{p}_1 \cdot \vec{p}_3)\, BW_{R,12} \ , \tag{4}$$

where F_{P,Rd_3} and $F_{R,d_1 d_2}$ are form factors, and the three-momenta $\vec{p}_1$ and $\vec{p}_3$ are evaluated in the resonance rest frame. The explicit momentum dependence in (4) follows from Eq. (2), just assuming Lorentz invariance and summing over all possible polarizations of the intermediate vector meson resonance. The differential decay width of this reaction can be written as

$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}|^2 dm_{12}^2\, dm_{13}^2 \ , \tag{5}$$

where $m_{ij}^2 = (p_i + p_j)^2$ and M is the mass of the decaying meson P.

For the decays considered here, there is a strong suppression at the resonant peak, i.e. when $m_{12}^2 \simeq m_0^2$. This suppression is due to purely kinematic effects. To see this, let us take $\mathcal{A} \simeq$ constant for a given value of m_{12}^2 —which means to neglect the dynamics of the decay— and integrate over the variable m_{13}^2 within the kinematic limits of the three body phase space. It is easy to see that

$$\frac{d\Gamma}{dm_{12}^2} \propto |\mathcal{A}|^2\, |\vec{p}_1| \ , \tag{6}$$

where $\vec{p}_1$ is the three-momentum of d_1 in the resonance rest frame. Since we are assuming that the mass of the resonance is just above the threshold, $m_0 \simeq m_1 + m_2$, at the resonance peak both particles d_1 and d_2 will be produced almost at rest. Thus Eq. (6) implies a suppression in the partial width.

The effect is even stronger in the particular case of a vector resonance. In that case, as stated in Eq. (4), the amplitude is proportional to $\vec{p}_1 \cdot \vec{p}_3$. Assuming that the form factors are slow-varying functions of the phase space variables, the partial width is finally expected to be suppressed by a factor $(|\vec{p}_1|/\Lambda)^3$, where Λ is some natural scale of the process, typically of order M.

Eqs. (4) and (6) show up the main point of this Letter. The decay width $\Gamma(P \to d_1 d_2 d_3)$ is driven by two *competitive* effects. On the one hand, the BW propagator strongly enhances the decay amplitude in the vicinity of the resonance mass. On the other hand, kinematic effects suppress the differential width at the resonance peak, hence the decay rate for other kinematic regions is comparatively enhanced. The usual suppression of the differential decay width for $P \to R d_3 \to d_1 d_2 d_3$ outside the window allowed by the BW function is not obvious in this case.

To clarify our point, let us emphasize that we are not claiming that there is an enhancement of the resonant *production* probability $P \to R d_3$ outside the BW peak. On the contrary, the point is that due to the suppression of the resonant *decay rate* $R \to d_1 d_2$ at the resonance central mass, the *combined* production + decay probability within and outside the peak could be of comparable orders. In other words, the total width (i.e., integrated over the whole phase space) is indeed small; the important point is that, even if the decay proceeds through a BW-described resonance, the relative weight for the decay rate near or far from the resonance mass shell is not just driven by the BW function.

In order to show that this effect is not a simple academic thought, we present in the following a theoretical estimate for an actual process. Let us consider the decay $B^+ \to D^{*0} D_s^+; D^{*0} \to D^0 \pi^0$. Using Eq. (4), it is possible to get an estimate for the differential decay rate $d\Gamma$ as a function of m_{12}^2. Since the form factors are usually smooth functions, we will assume as a first guess that they are constant (experimental analyses show that form factor shapes have no significant effect on the total systematic error of a given Dalitz plot fit [9]). In this way we can calculate the ratio

$$r = \frac{\displaystyle\int_{m_{12}^2=(m_0-n\,\Gamma)^2}^{m_{12}^2=(m_0+n\,\Gamma)^2} |\mathcal{A}|^2 d\Phi}{\displaystyle\int |\mathcal{A}|^2 d\Phi} , \tag{7}$$

where $d\Phi$ is an element of the three body phase space, m_0 is the D^{*0} resonance mass ($m_0 = 2007$ MeV), Γ is the D^{*0} width, and n is a real number. Γ is presently unknown, its upper limit being 2.1 MeV with a 90% confidence level [1]. The integral in the denominator is calculated over the whole phase space, while that in the numerator is limited to a window in m_{12}^2. Thus, r is a measure of the relative number of events that are expected to fall within the resonance peak.

We quote in Table I the values of r for some input values of n and Γ. Our results show that the quantum effect we are describing can be very strong if the resonance width is as large as 2 MeV, and it remains quite significant even if Γ is of the order of 0.1 MeV. For comparison, we include in the last column the values of r corresponding to a fictitious resonance having the same quantum numbers as D^{*0} but a higher mass, $m_0 = 2.6$ GeV. This particle would not suffer the kinematic suppression in its decay to $D^0\pi^0$, consequently a small value of n is enough to get r above 90%. We have found that in this case the results for r are independent of the specific value of Γ.

Figures 1a and 1b show the kinematic distribution of the events for the three body decay $B^+ \to D^0 \pi^0 D_s^+$, assuming that the process is dominated by the $D^{*0}(2007)$ resonance channel [10] and considering a D^{*0} width $\Gamma = 1$ MeV. The plots correspond to a Monte Carlo simulation of 10000 events, performed using Eqs. (4) and (5). Figure 1a is the Dalitz plot of the decay as a function of the invariant masses $m^2_{D^0\pi^0}$ and $m^2_{D_s^+\pi^0}$, while in Figure 1b we represent an histogram of the number of events as a function of $m^2_{D^0\pi^0}$.

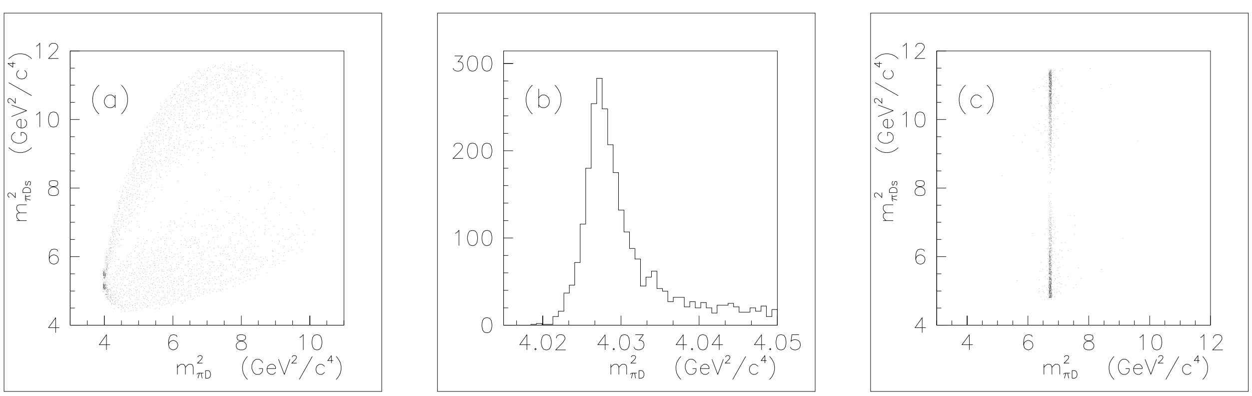

FIG. 1. Simulation of the decay $B^+ \to D^{*0} D_s^+$; $D^{*0} \to D^0 \pi^0$, with $\Gamma = 1$ MeV and 10000 generated events. (a) is the Dalitz plot in the plane $[(p_{D^0}+p_{\pi^0})^2,\, (p_{D_s^+}+p_{\pi^0})^2]$, whereas (b) is a projection on the $(p_{D^0}+p_{\pi^0})^2$ axis. (c) is same as (a), with the D^{*0} mass shifted to 2.6 GeV

It is seen that the Dalitz plot shape in Fig. 1a is quite different from that naively expected for a decay mediated by a $\Gamma = 1$ MeV vector resonance. This becomes evident by looking at Figure 1c, where we show a Monte Carlo simulation of the same process, now shifting the D^{*0} mass to the fictitious value of 2.6 GeV considered in Table I. The striking difference between the event distribution in both plots arises from the kinematic suppression discussed above. However, the situation displayed in Fig. 1a can be misleading if one just looks at the events for which $m^2_{D^0\pi^0}$ is near the D^{*0} mass. In fact, despite the spreading of events along the whole phase space, the event density is still much larger in the peak region than anywhere else. Therefore, Figure 1b could be mistaken for a $\Gamma = 1$ MeV Breit-Wigner function, with some background in the right sideband [11]. It would be then natural to apply the usual method, that means to consider that the events within a window of a few Γ around the peak amount almost the total number of decay events. But, according to our simulation, this would be wrong by a factor as large as 3 to 4 (see Table I). Indeed, even if the amplitude is peaked around the resonance mass, the phase space area outside the peak is comparatively so large that the number of events falling in this region becomes very important.

It is important to stress that our estimates rely on two basic assumptions. First, we have taken the form factors to be approximately constant along the phase space. Second, we have assumed that the Breit-Wigner shape remains valid far beyond a small region around the peak. Whereas the first hypothesis is quite natural and should not have a significant effect on our results, the second assumption can be hardly supported from the theoretical point of view. Our simulations are in this sense strongly model dependent, and this has to be kept in mind when looking at the numerical values presented in Table I.

In any case, our simulations can be taken as a severe warning, indicating that many reported results can be spoiled by the quantum effect described in this Letter. This includes the analysis of the decays $B \to D^{*0} d_3$, $B \to D^{*+} d_3$ and $B \to D_s^{*+} d_3$, with $d_3 = \pi^0, \eta, K, D_s^+, D^0$, etc. These channels have been measured [12] using $D^0\pi^0 d_3$ —respectively $D^+\pi^0 d_3$ and $D_s^+\pi^0 d_3$— as final states, and imposing a cut in the invariant mass $m^2_{D^0\pi^0}$ —respectively $m^2_{D^+\pi^0}$ and $m^2_{D_s^+\pi^0}$. According to the simple analysis presented here, in all these cases the effect would be of the order predicted in Table I. In particular, in the case of the D^{*+} resonance, whose width has recently been reported to be 0.1 MeV, the effect is expected to be of the order of 30%. However, it is important to take into account that the technique used to measure Γ precisely assumes [8] that all relevant events lie just within a small window around the resonance central mass. Thus, the actual D^{*+} width is possibly larger.

Finally, let us mention that ϕ meson production and decay measurements could also be affected by the quantum effect described above. Since the ϕ resonance can be produced in D meson decays, a significant amount of data is presently available, and many Dalitz plot analyses have already been done. However, notice that the threshold of the decay $\phi \to KK$ is about 25 MeV away from the ϕ mass, so that the available phase space is not as narrow as in the D^* case. Performing a Monte Carlo simulation for the decay $D \to \phi\pi$ similar to that described above for the process $B \to D^* D_s$, we have found that the expected effect could be in this case as large as 40%.

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[9] See for example E687 Collab., P.L. Frabetti *et al.*, Phys. Lett B **331**, 217 (1994).
[10] In fact, the decay can also proceed through an intermediate resonance D_s^{*+}, as well as other higher resonant states. The inclusion of these contributions in our analysis does not affect significantly the results.
[11] See for example Figure 1 of Ref. [7].
[12] The list is very extensive. For a complete review see [1]. A very recent measurement is that of Belle Collab., K. Abe *et al.*, hep-ex/0107048.

n	0.1 MeV	0.5 MeV	1 MeV	2 MeV	fictitious
1	61%	30%	18%	11%	70%
3	69%	36%	24%	17%	90%
10	73%	41%	30%	25%	97%
30	75%	46%	37%	36%	99.5%

TABLE I. The ratio r as explained in the text, for different values of n (1 to 30) and Γ (0.1 to 2 MeV). The last column shows the values of r for a fictitious resonance with mass 2.6 GeV —see text.